MLQ 2021

Improving Quantum Sensing with Variational Methods

JOHANNES JAKOB MEYER, FU BERLIN & QMATH

@jj_xyz

arxiv:2006.06303

A variational toolbox for quantum multi-parameter estimation

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(Dated: June 11, 2020)

Johannes Borregaard
TU Delft

Jens Eisert
FU Berlin

Quantum Metrology

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Physical quantities (magnetic fields, energies, …)
need to be **measured** accurately

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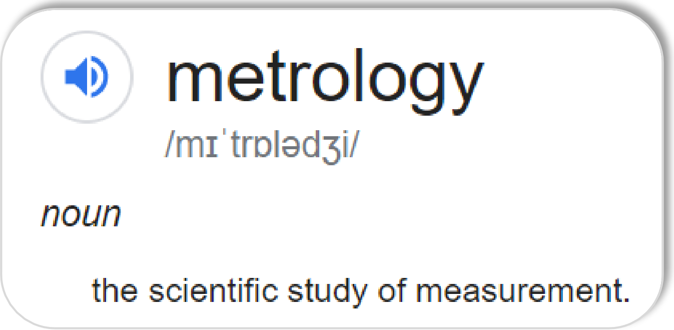
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noun
the scientific study of measurement.

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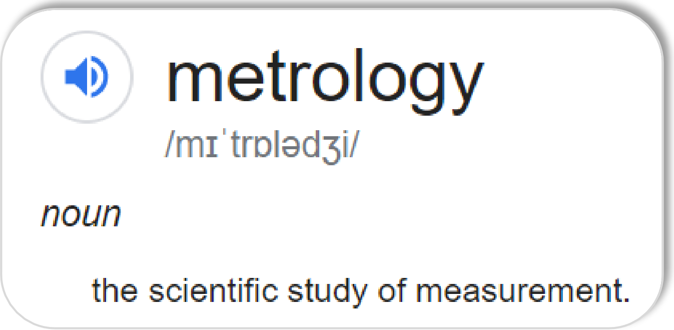

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Probe
State

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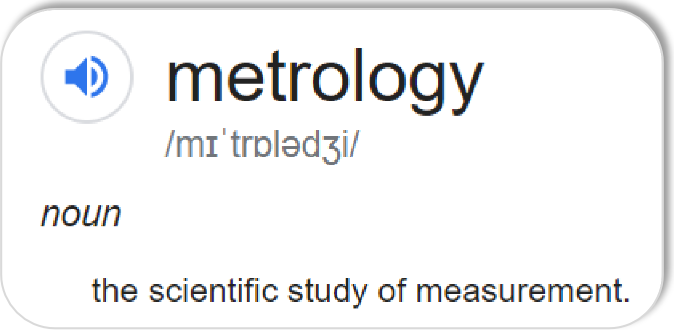

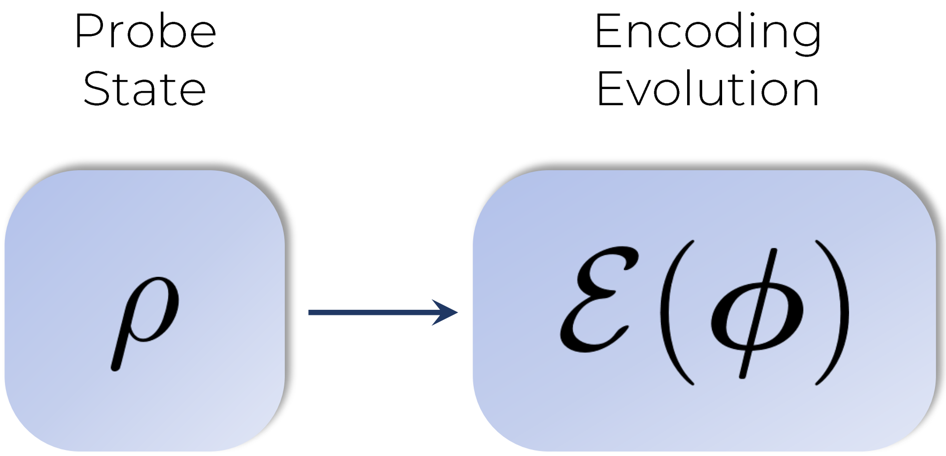

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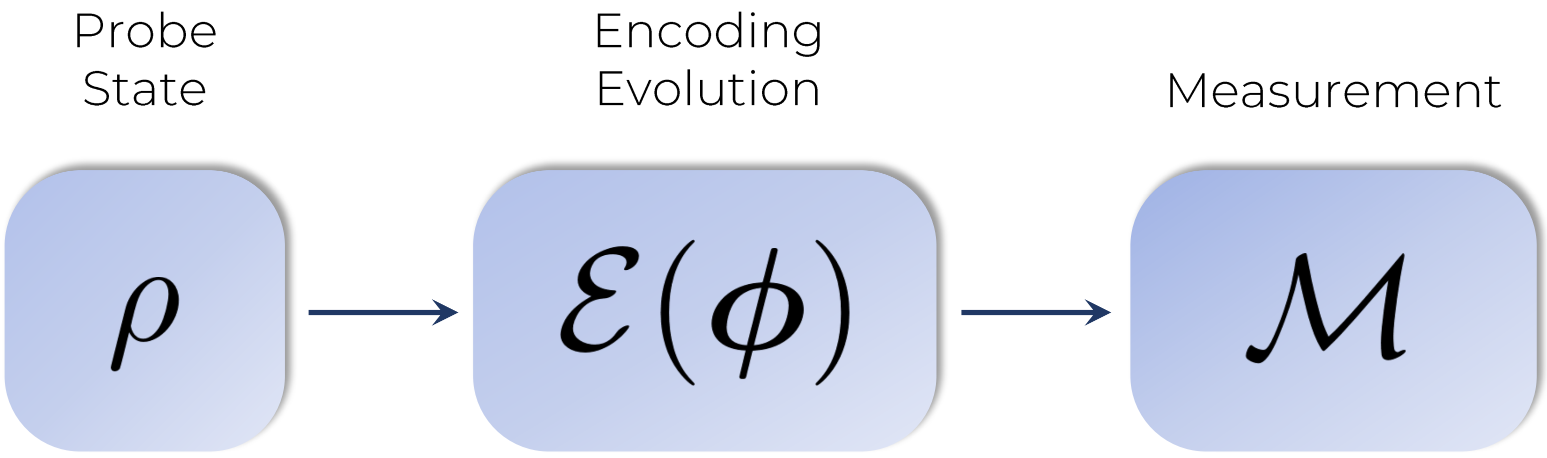

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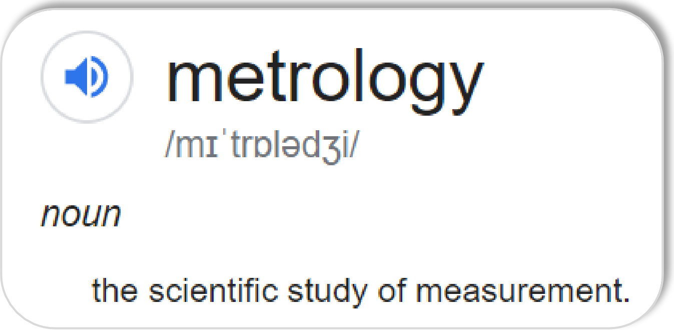

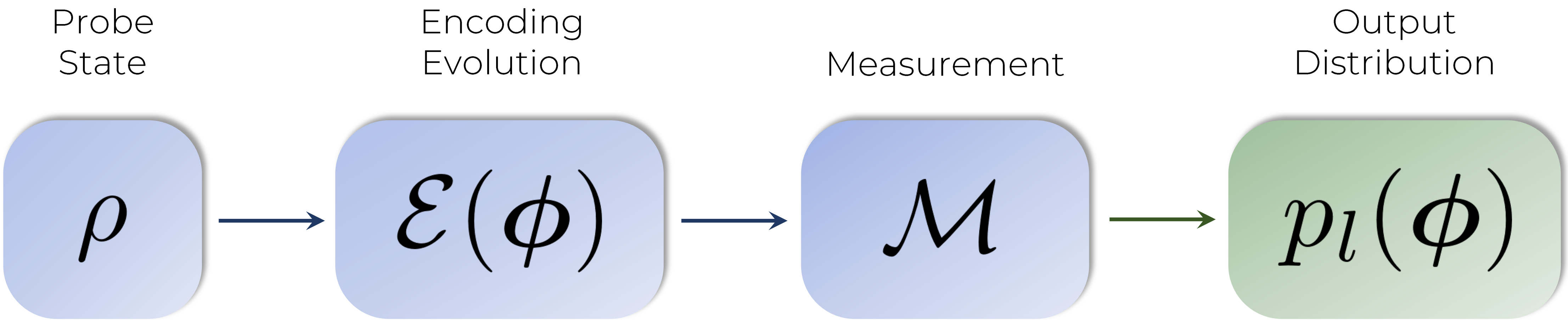

Gathering Intuition

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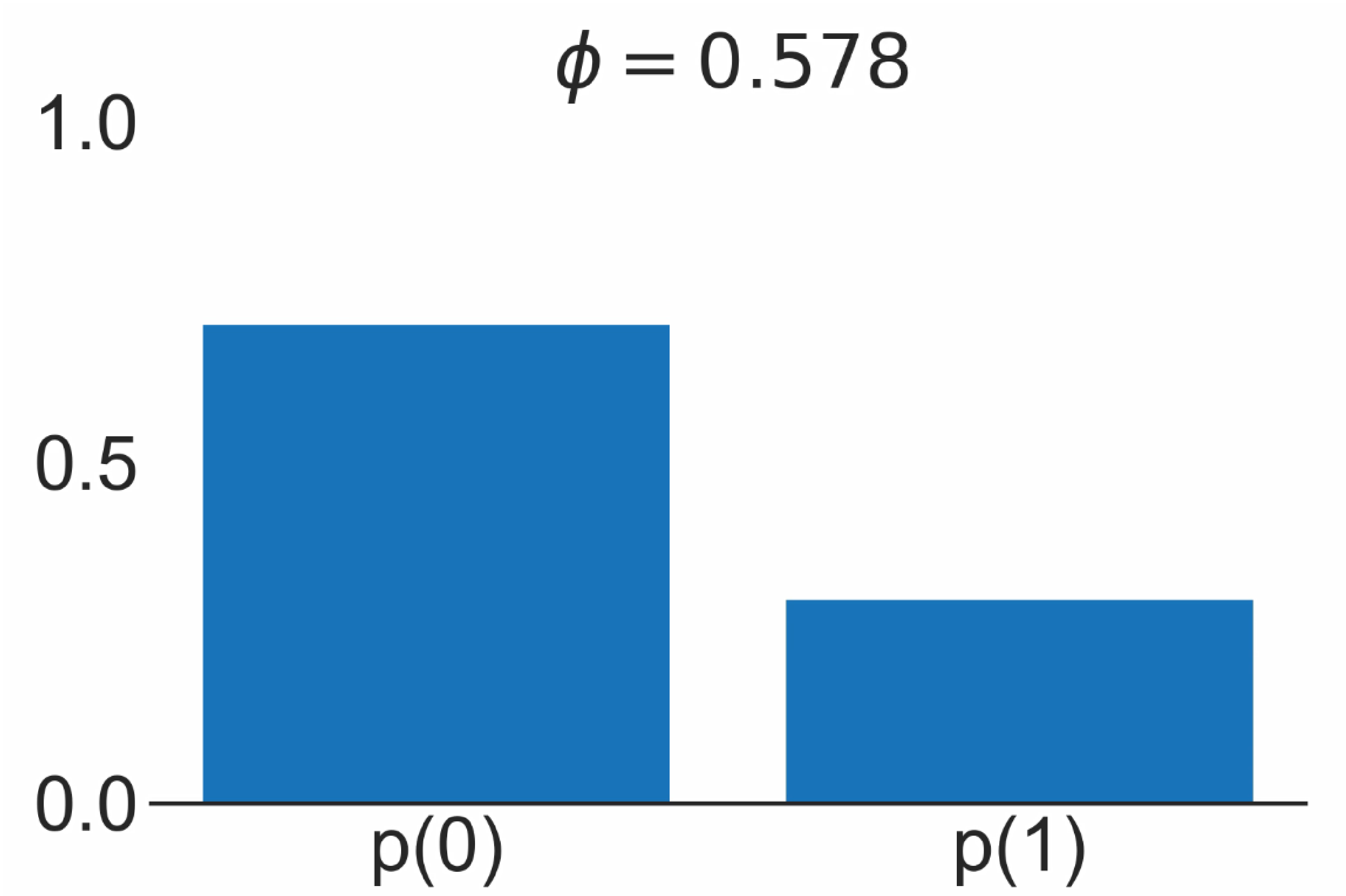

Measuring Performance

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Task: Compute an **estimator** from the output distribution

$$p_l(\boldsymbol{\phi}) \;\rightarrow\; \hat{\boldsymbol{\varphi}} \colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

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→ Classical Fisher information should be used to judge sensing quality!

Optimal Metrology

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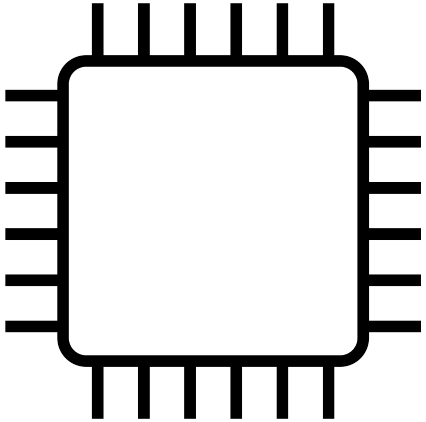

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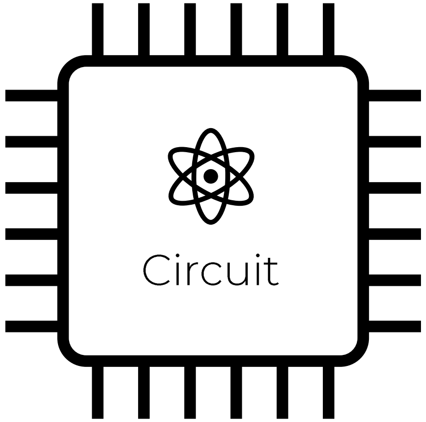

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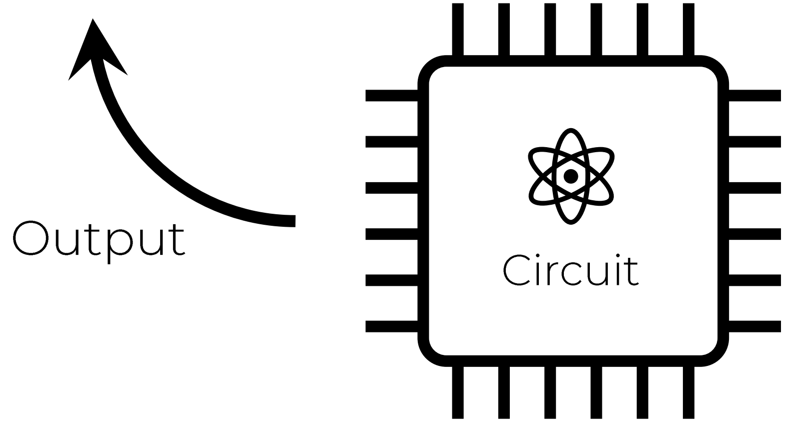

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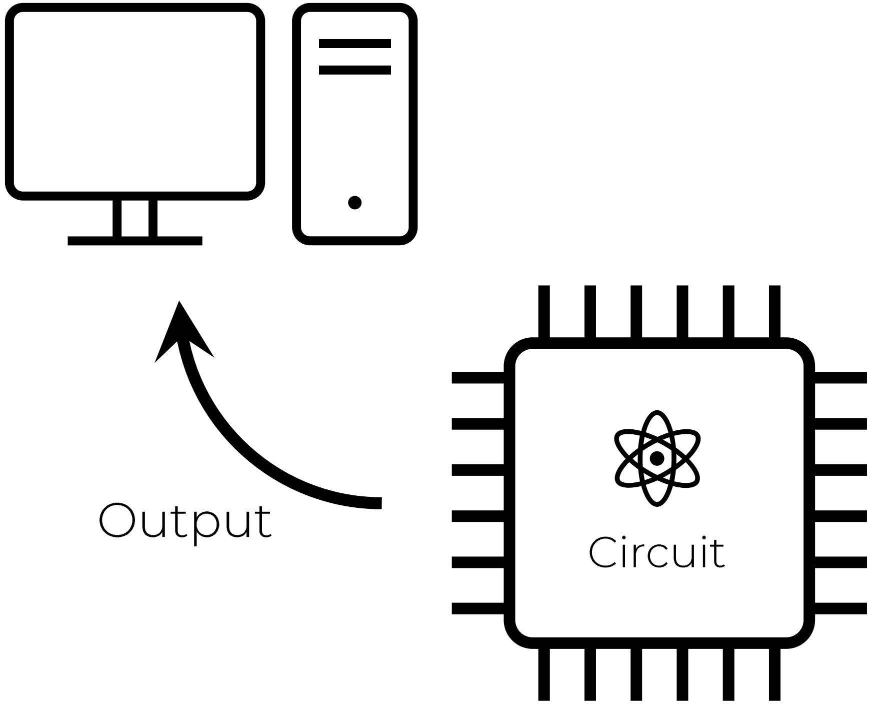

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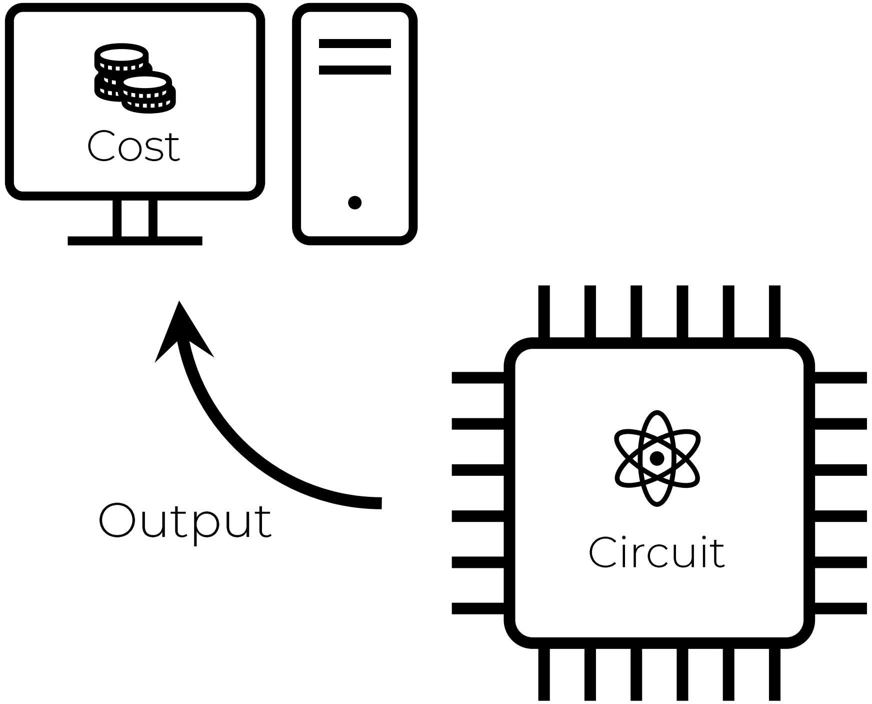

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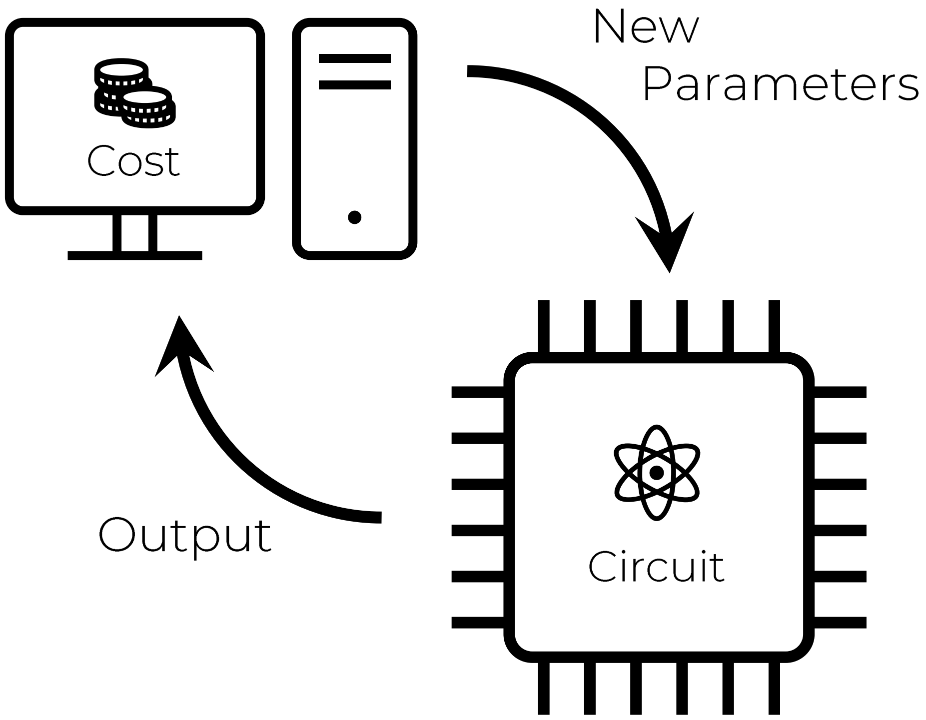

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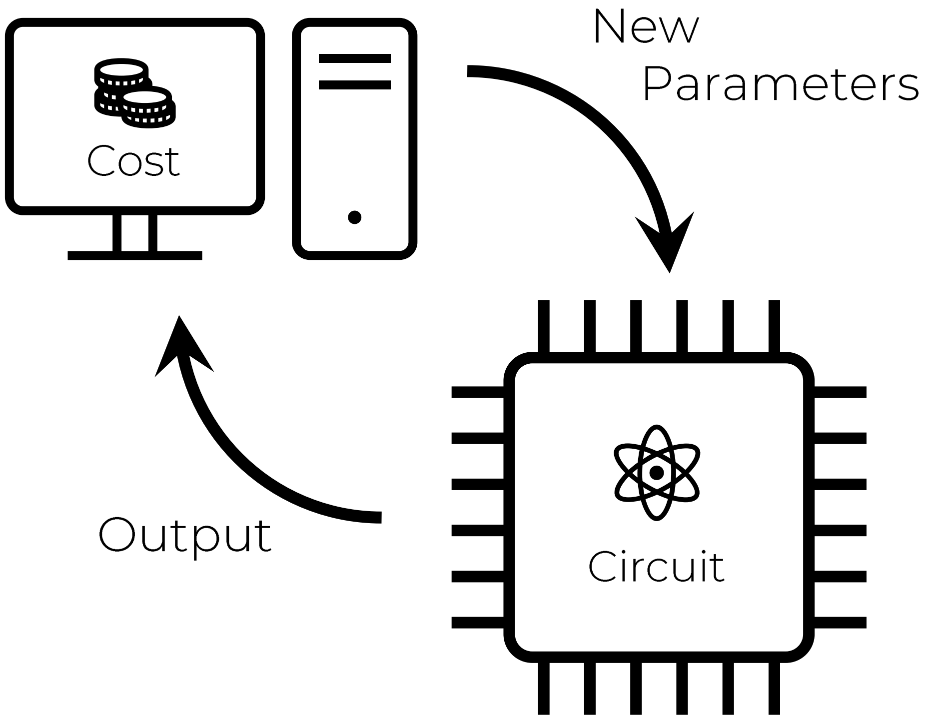

Prior work[1,2] focused on single-parameter metrology and surrogates for the Quantum Fisher Information

[1]Kaubrügger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).

Cost Function

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Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

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Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

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$$\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{f}})\} \geq \frac{1}{n}\,\mathrm{Tr}\{W I_{\boldsymbol{f}}^{-1}\} = \frac{1}{n} C_W$$

Calculation of Fisher Information

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Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_l \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$

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Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2} \left[p_l \left(\boldsymbol{\phi} + \frac{\pi}{2} \boldsymbol{e}_j \right) - p_l \left(\boldsymbol{\phi} - \frac{\pi}{2} \boldsymbol{e}_j \right) \right]$$

[1]Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2]Banchi, Leonardo, and Gavin E. Crooks. arXiv preprint arXiv:2005.10299 (2020).

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The classical Fisher information matrix w.r.t. post-processed parameters can be computed using the post-processing‘s Jacobian

The cost function is obtained from a weighted trace of the Cramér-Rao bound

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Multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

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Details on the implementation of parameter-shift rules in experiments

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Details on the implementation of parameter-shift rules in experiments

Numerical experiments that showcase the performance of the approach

Take-Home Message

> Variational methods on near-term quantum computers can be used to improve quantum sensors

Thank you for your attention!

Paper

Demo

Slides

www.johannesjakobmeyer.com

@jj_xyz

The Algorithm Landscape

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Single Parameter

Multiparameter

The Algorithm Landscape

The Algorithm Landscape

Single Parameter

Multiparameter

Kaubrügger et al.

COST FUNCTION
Spin Squeezing

STATE PREPARATION
Fixed Circuit

MEASUREMENT
Fixed

Koczor et al.

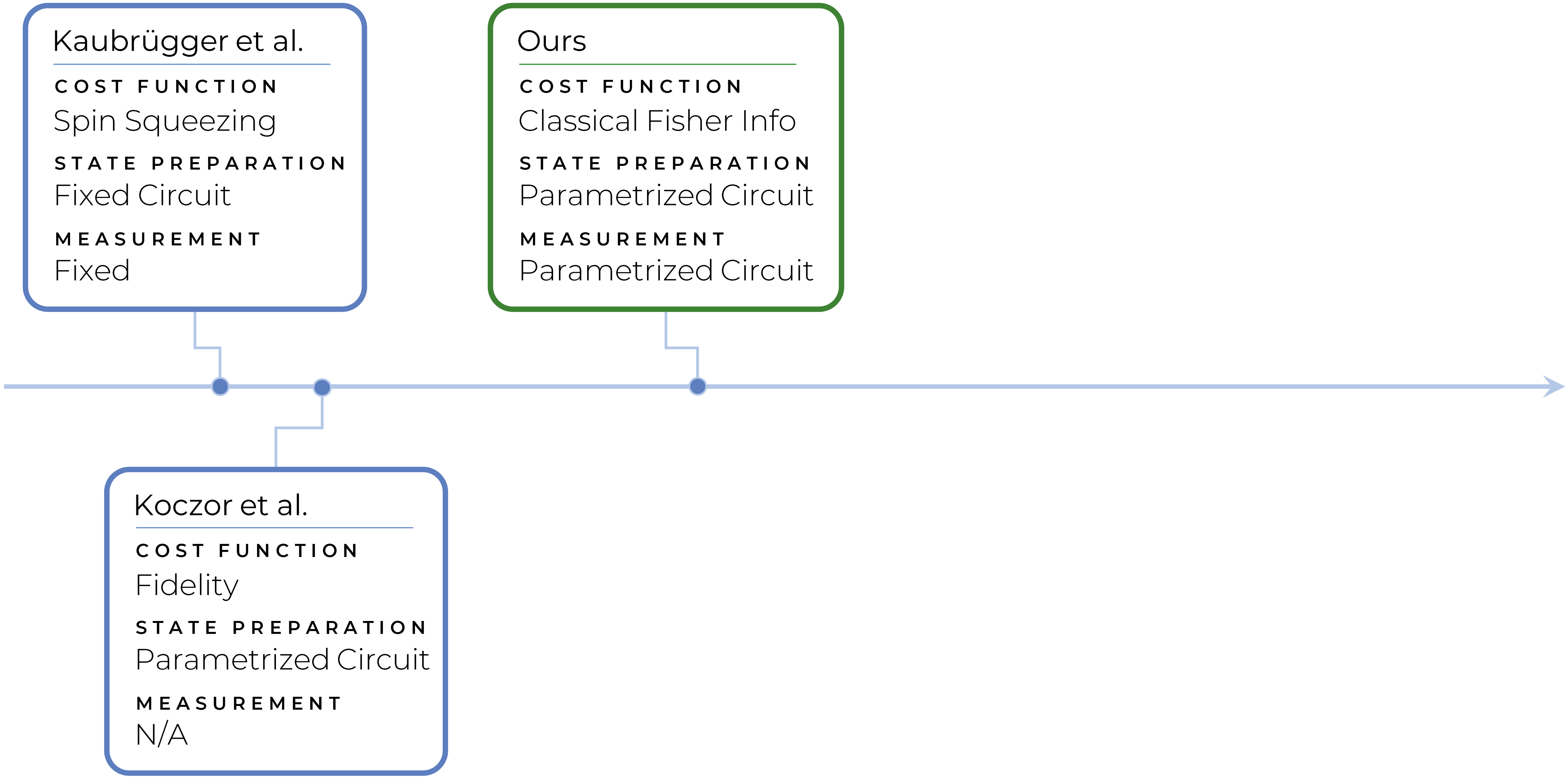
COST FUNCTION
Fidelity

STATE PREPARATION
Parametrized Circuit

MEASUREMENT
N/A

The Algorithm Landscape

Single Parameter Multiparameter



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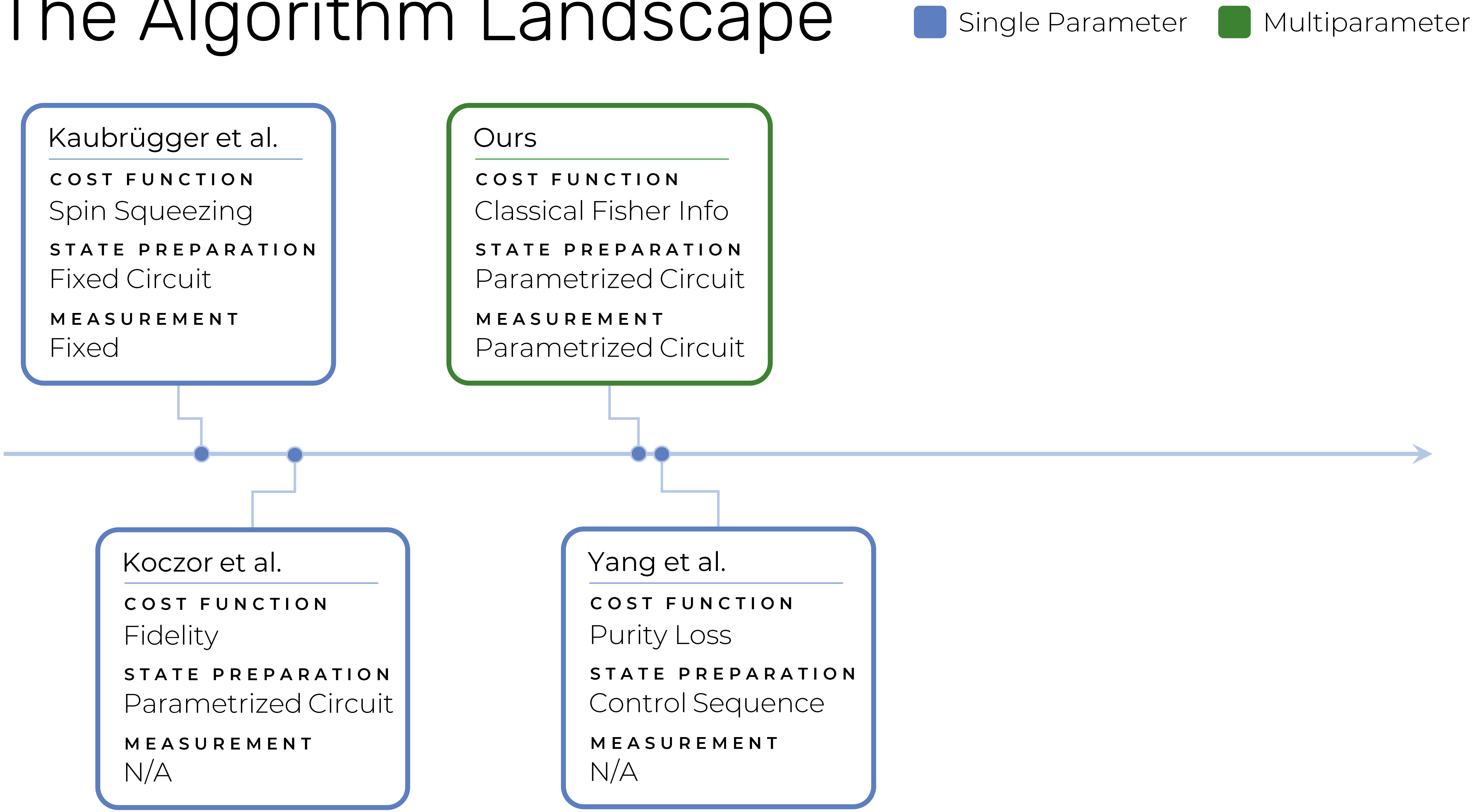

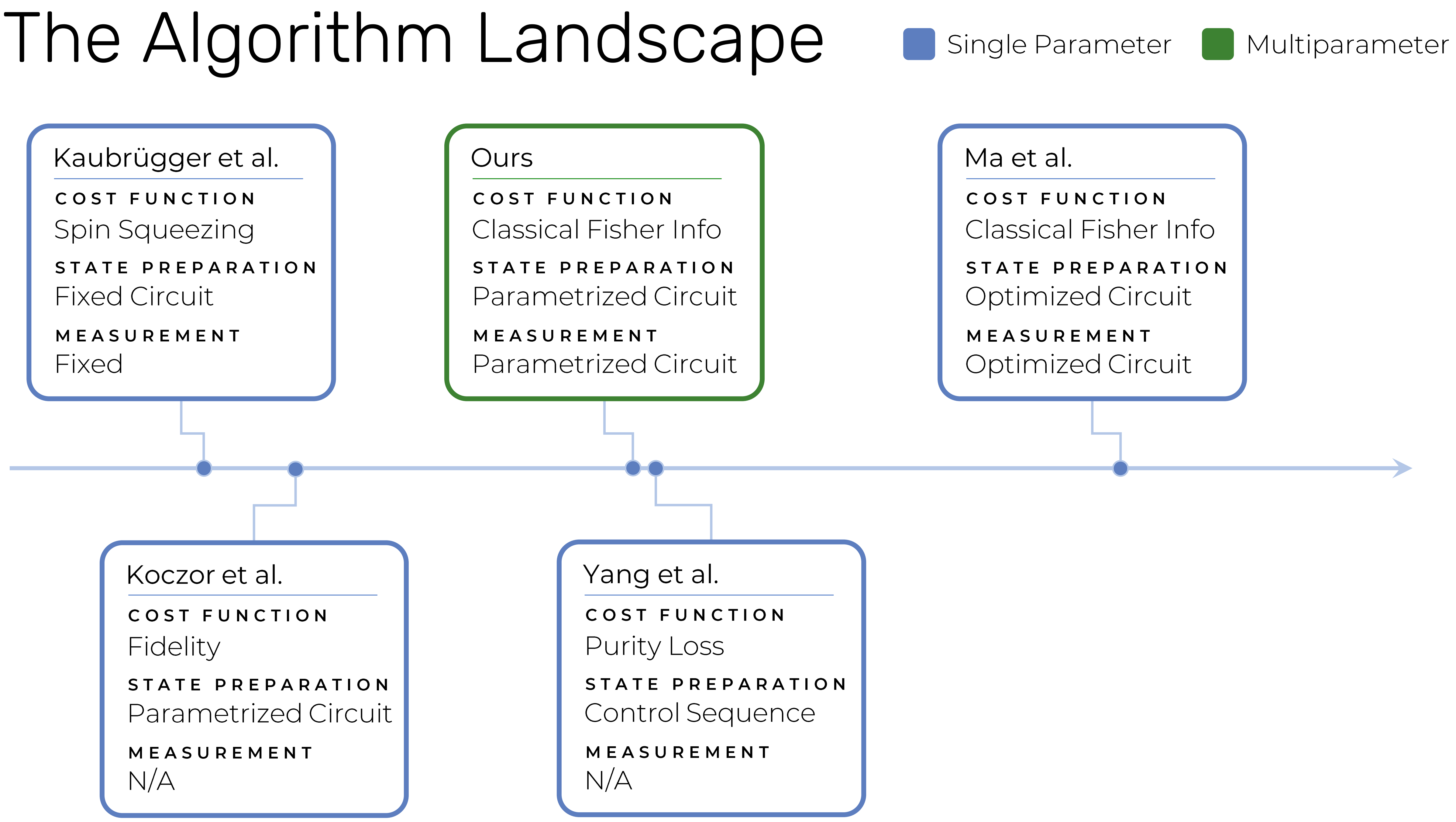
The Algorithm Landscape
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Multiparameter
Kaubrügger et al.
COST FUNCTION
Spin Squeezing
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Ours
COST FUNCTION
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Koczor et al.
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Fidelity
STATE PREPARATION
Parametrized Circuit
MEASUREMENT
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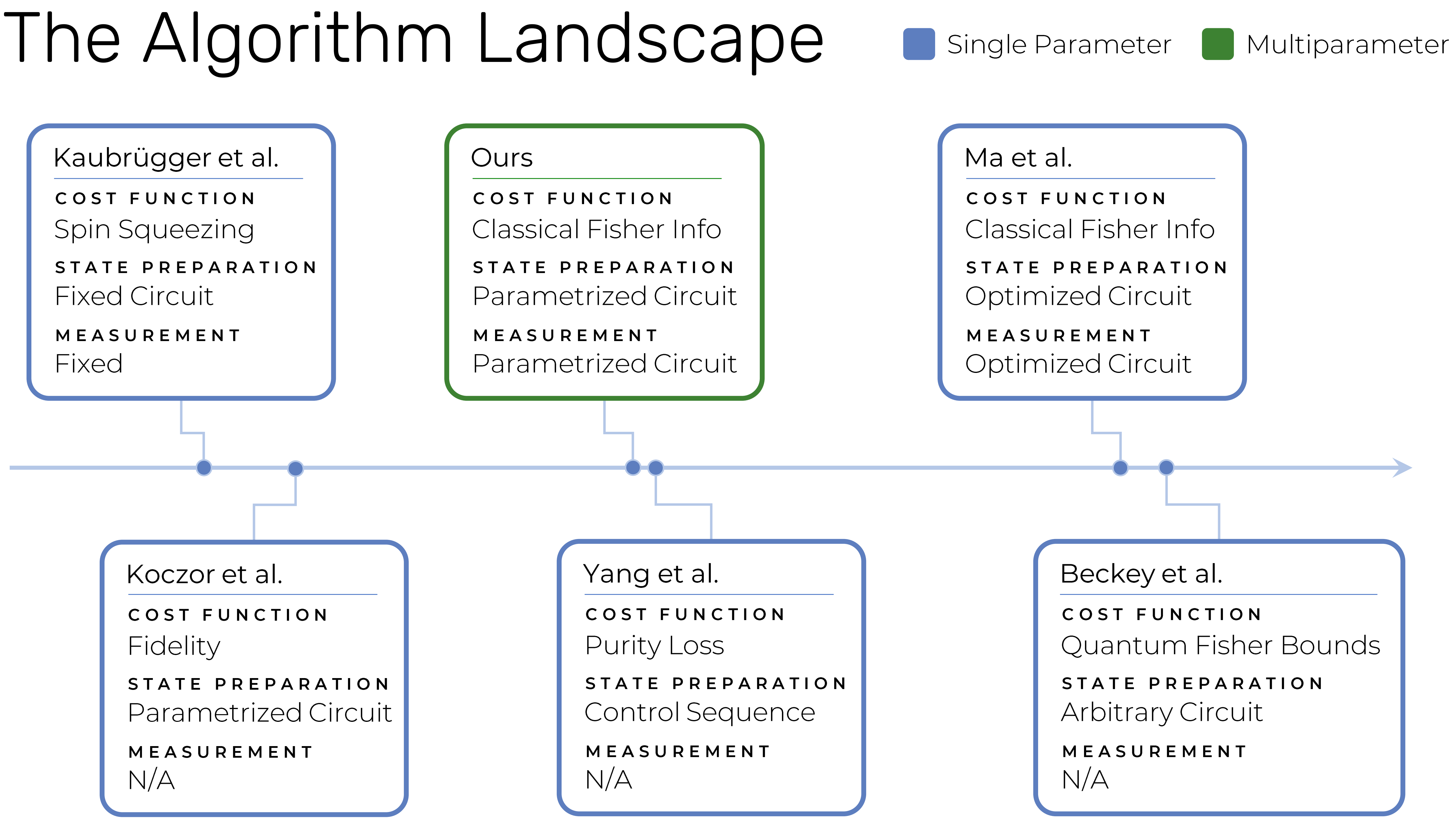
Yang et al.
COST FUNCTION
Purity Loss
STATE PREPARATION
Control Sequence
MEASUREMENT
N/A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STATE PREPARATION
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