YQIS 2021

Improving Quantum Metrology with Variational Methods

JOHANNES JAKOB MEYER, FU BERLIN & QMATH

@jj_xyz

arxiv:2006.06303

A variational toolbox for quantum multi-parameter estimation

Johannes Jakob Meyer,[1] Johannes Borregaard,[2,3] and Jens Eisert[1]
[1]*Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany*
[2]*Qutech and Kavli Institute of Nanoscience, Delft University of Technology, 2628 CJ Delft, The Netherlands*
[3]*Mathematical Sciences, Universitetsparken 5, 2100 København Ø, Matematik E, Denmark*
(Dated: June 11, 2020)

Johannes Borregaard
TU Delft

Jens Eisert
FU Berlin

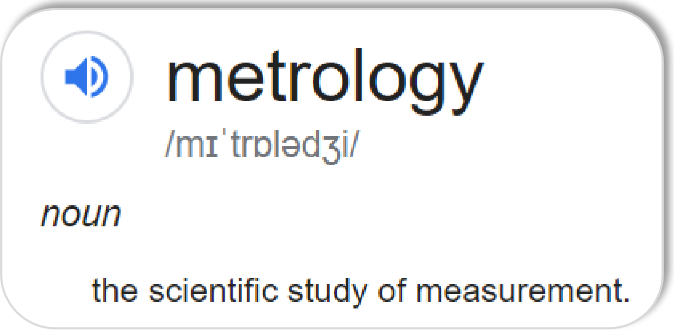
Quantum Metrology

Quantum Metrology

Physical quantities (magnetic fields, energies, ...)
need to be **measured** accurately

Quantum Metrology

Physical quantities (magnetic fields, energies, ...) need to be **measured** accurately



Quantum Metrology

Physical quantities (magnetic fields, energies, …) need to be **measured** accurately

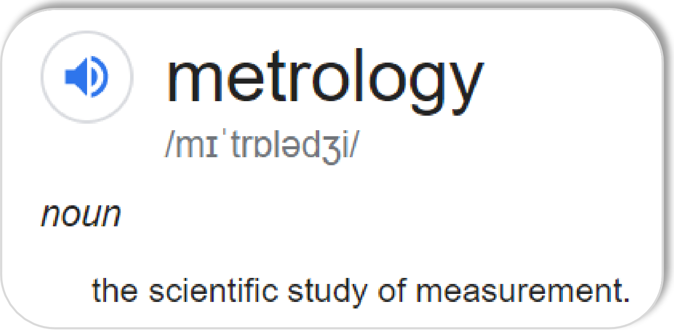
Study how **quantum effects** can help

metrology
/mɪˈtrɒlədʒi/
noun
the scientific study of measurement.

Quantum Metrology

Physical quantities (magnetic fields, energies, ...) need to be **measured** accurately

Study how **quantum effects** can help

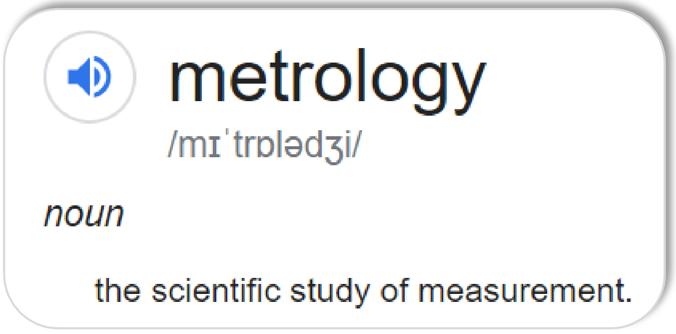


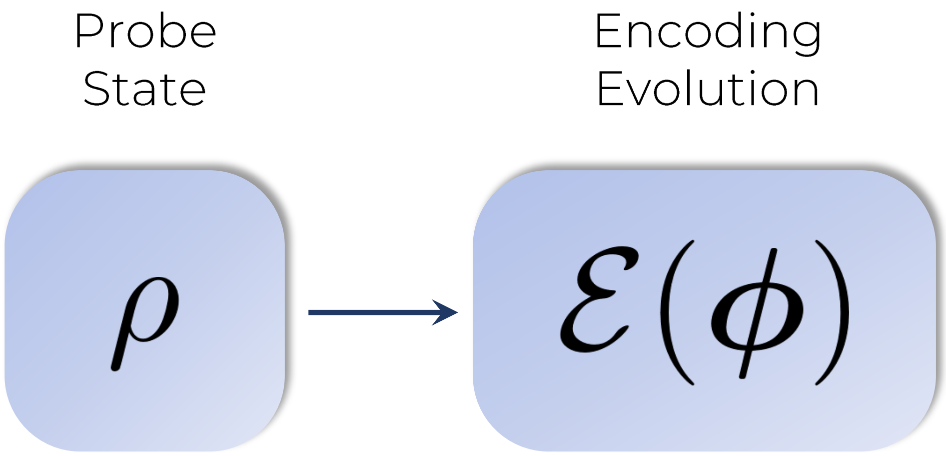
Probe
State

Quantum Metrology

Physical quantities (magnetic fields, energies, ...) need to be **measured** accurately

Study how **quantum effects** can help

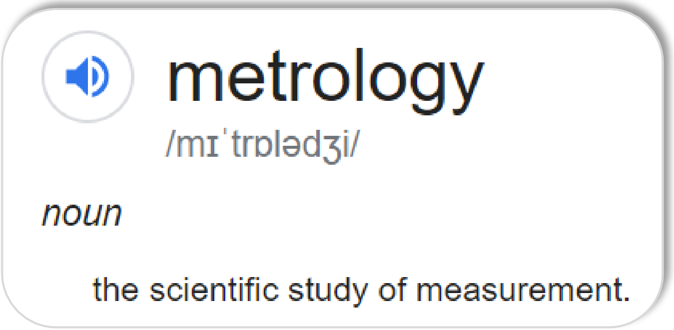


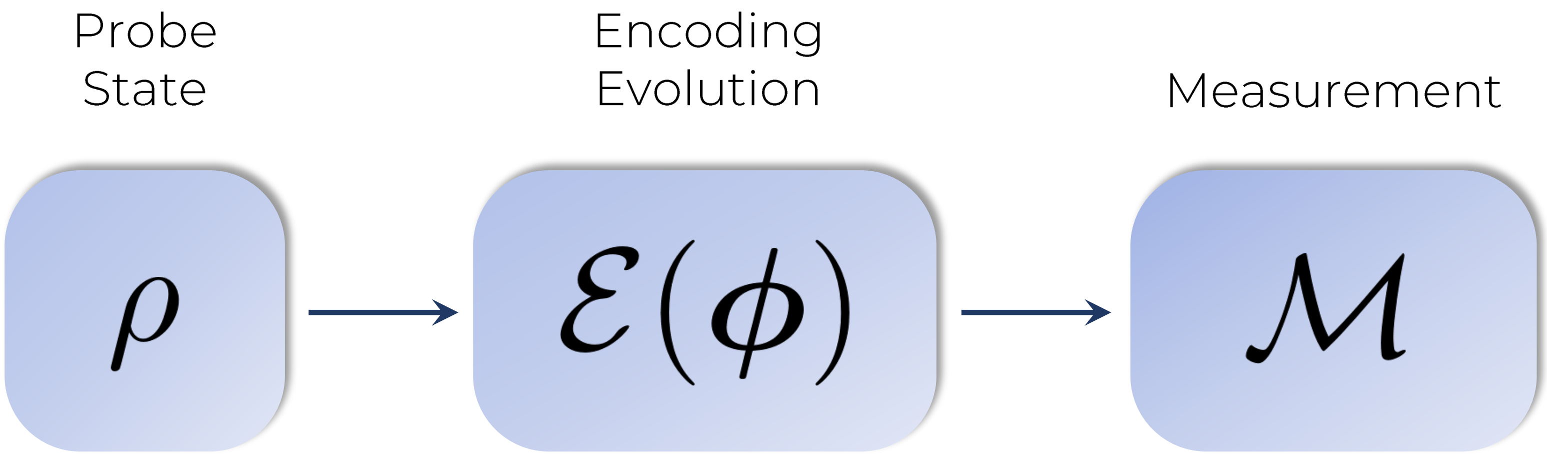


Quantum Metrology

Physical quantities (magnetic fields, energies, ...) need to be **measured** accurately

Study how **quantum effects** can help



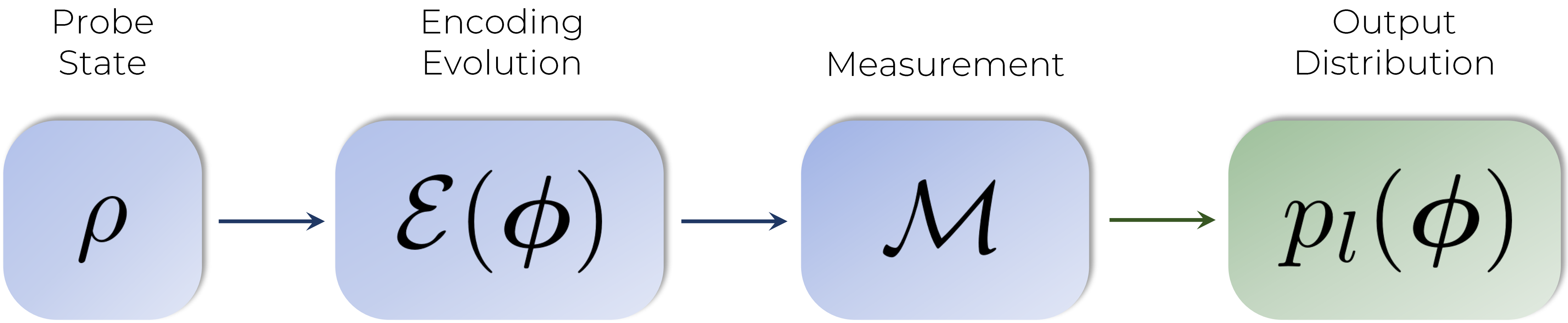


Quantum Metrology

Physical quantities (magnetic fields, energies, ...) need to be **measured** accurately

Study how **quantum effects** can help

metrology
/mɪˈtrɒlədʒi/
noun
the scientific study of measurement.



Gathering Intuition

Gathering Intuition

$$e^{-i\phi Z}$$

Gathering Intuition

$$|+\rangle \longrightarrow e^{-i\phi Z}$$

Gathering Intuition

$$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle / |-\rangle$$

Gathering Intuition

$$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle / |-\rangle$$

THEORY

Gathering Intuition

$$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle/|-\rangle$$

THEORY

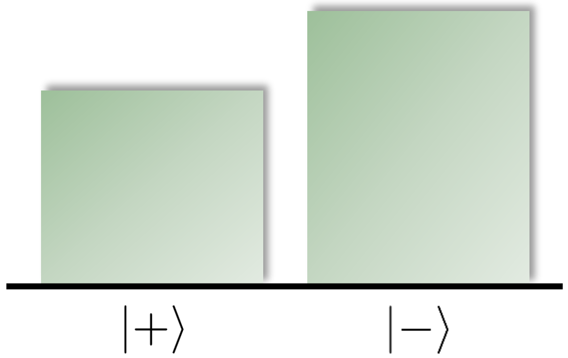
$$\phi = 0.9$$

Gathering Intuition

$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle / |-\rangle$

THEORY

$\phi = 0.9$

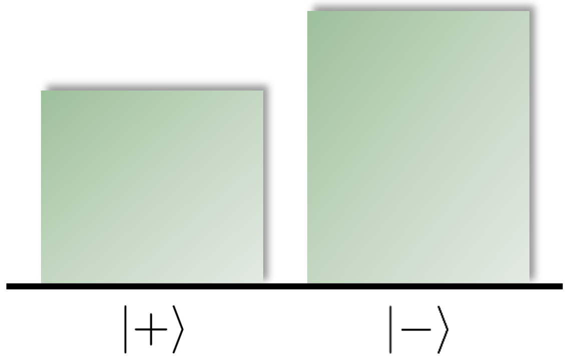


Gathering Intuition

$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle / |-\rangle$

THEORY

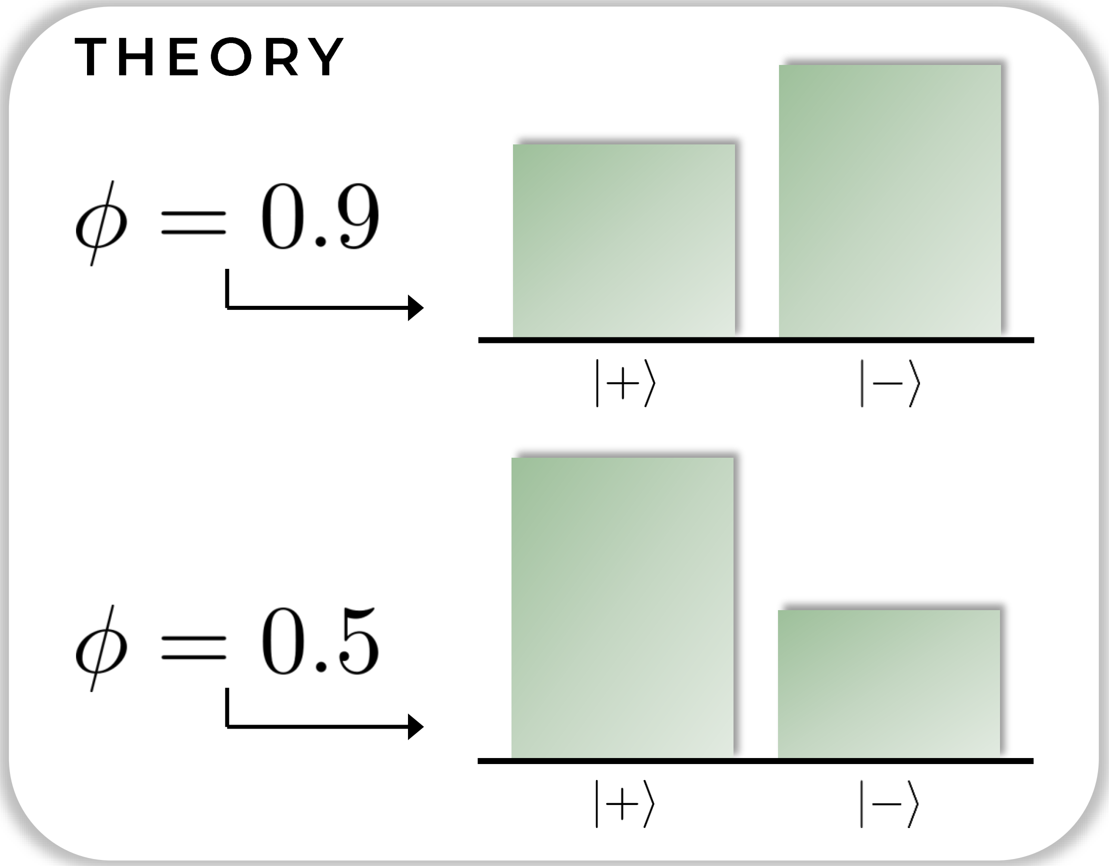
$\phi = 0.9$



$\phi = 0.5$

Gathering Intuition

$$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle / |-\rangle$$

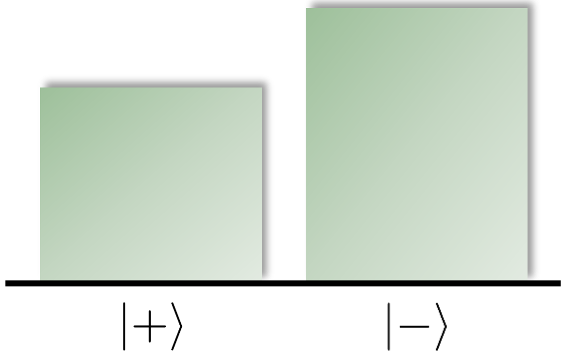


Gathering Intuition

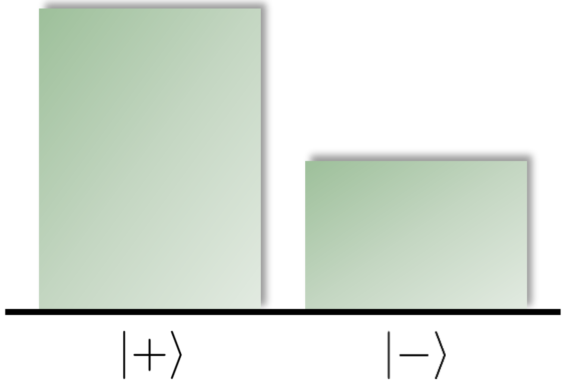
$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle/|-\rangle$

THEORY

$\phi = 0.9$



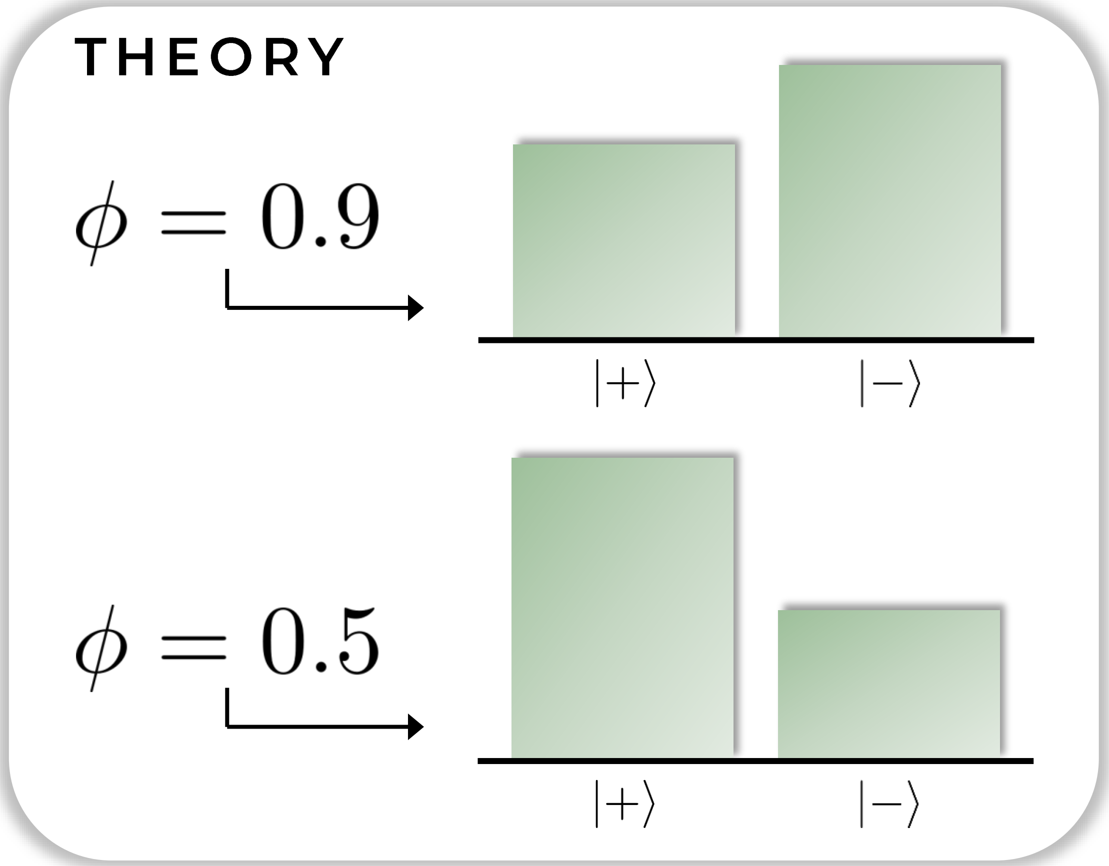
$\phi = 0.5$

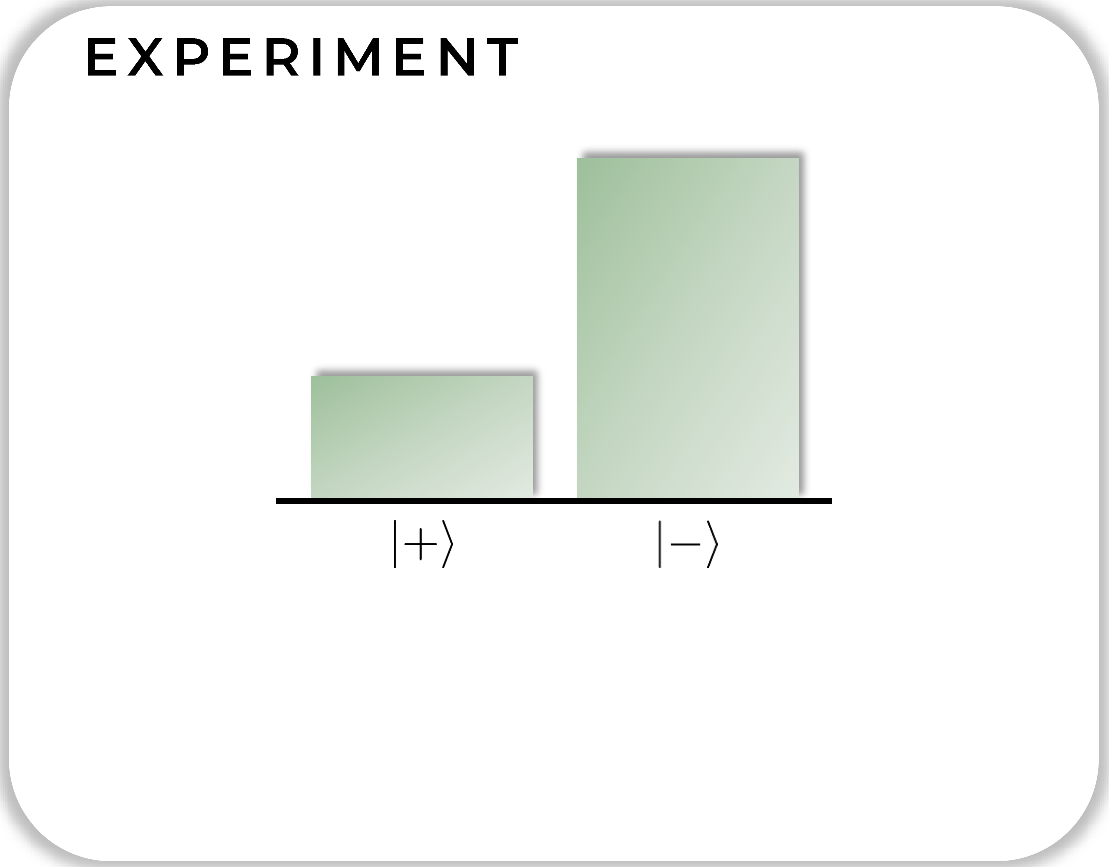


EXPERIMENT

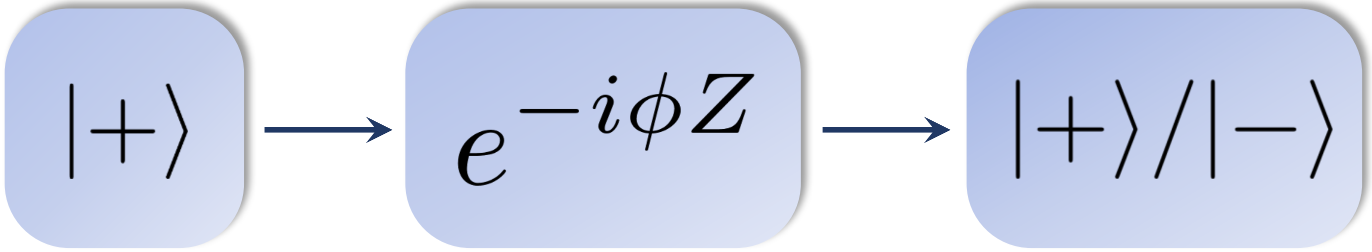
Gathering Intuition

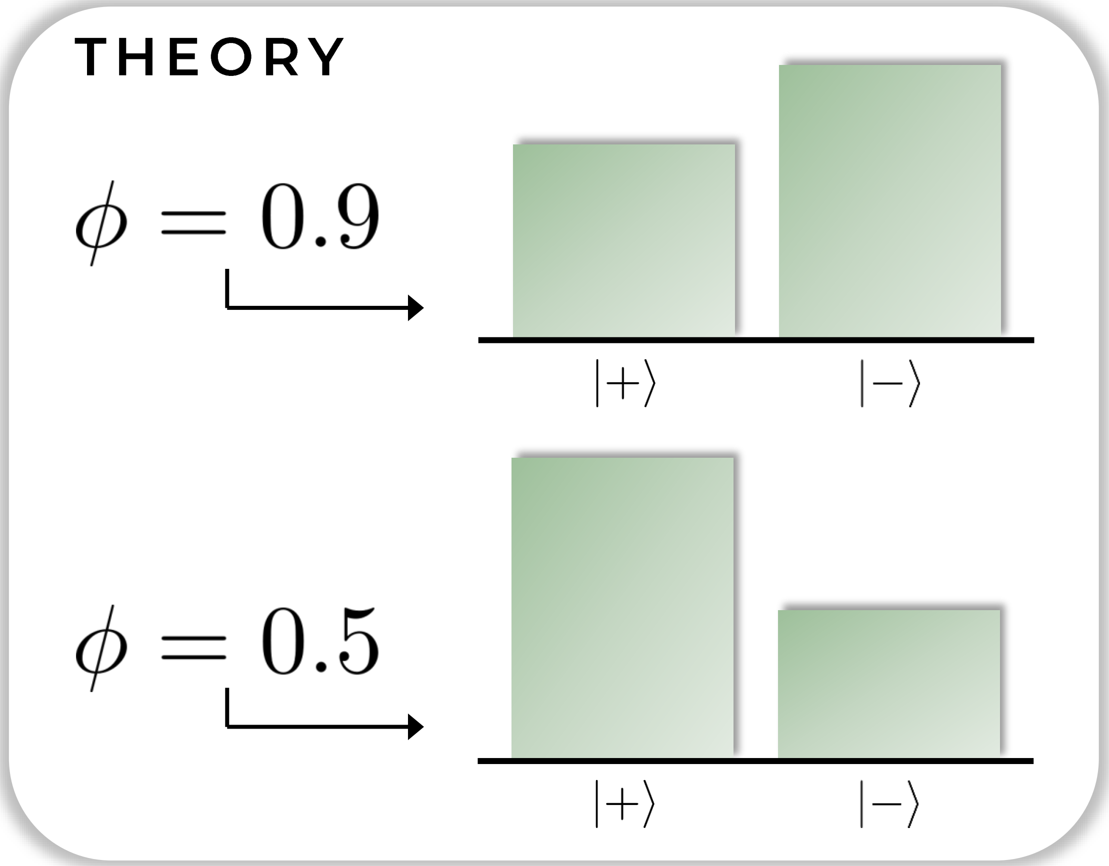
$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle / |-\rangle$

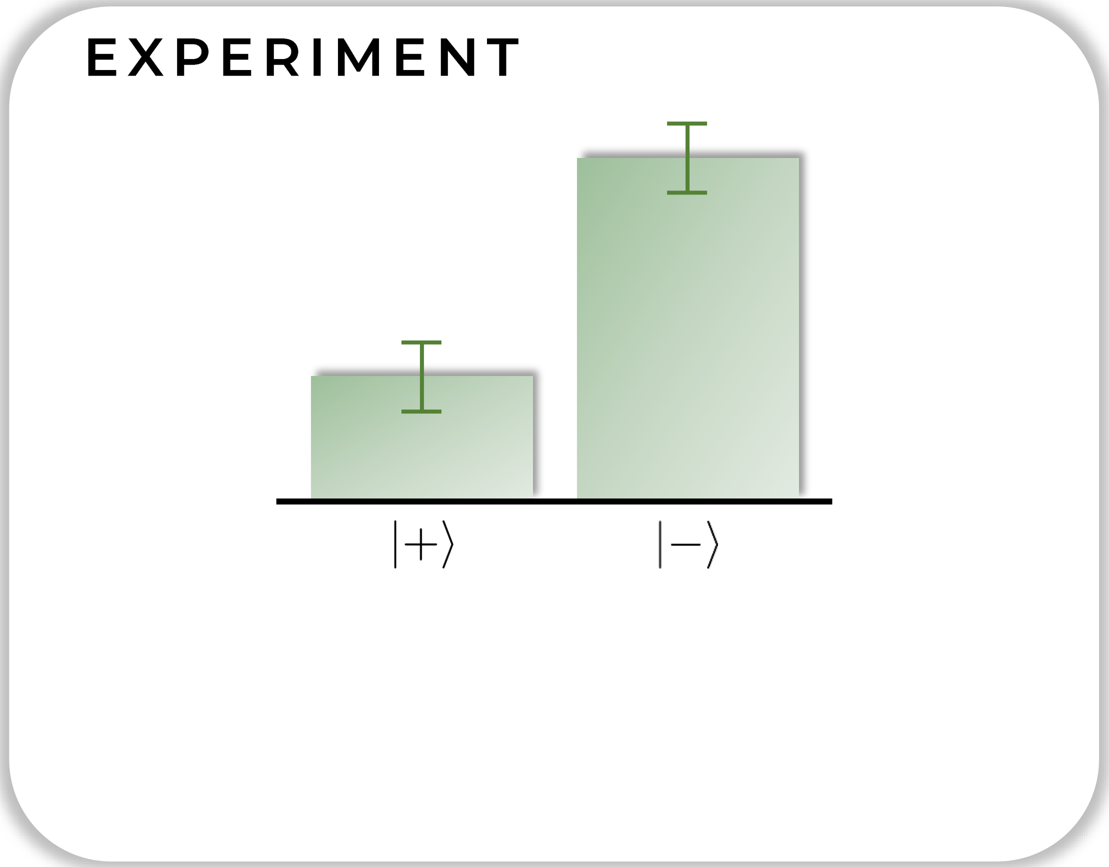




Gathering Intuition

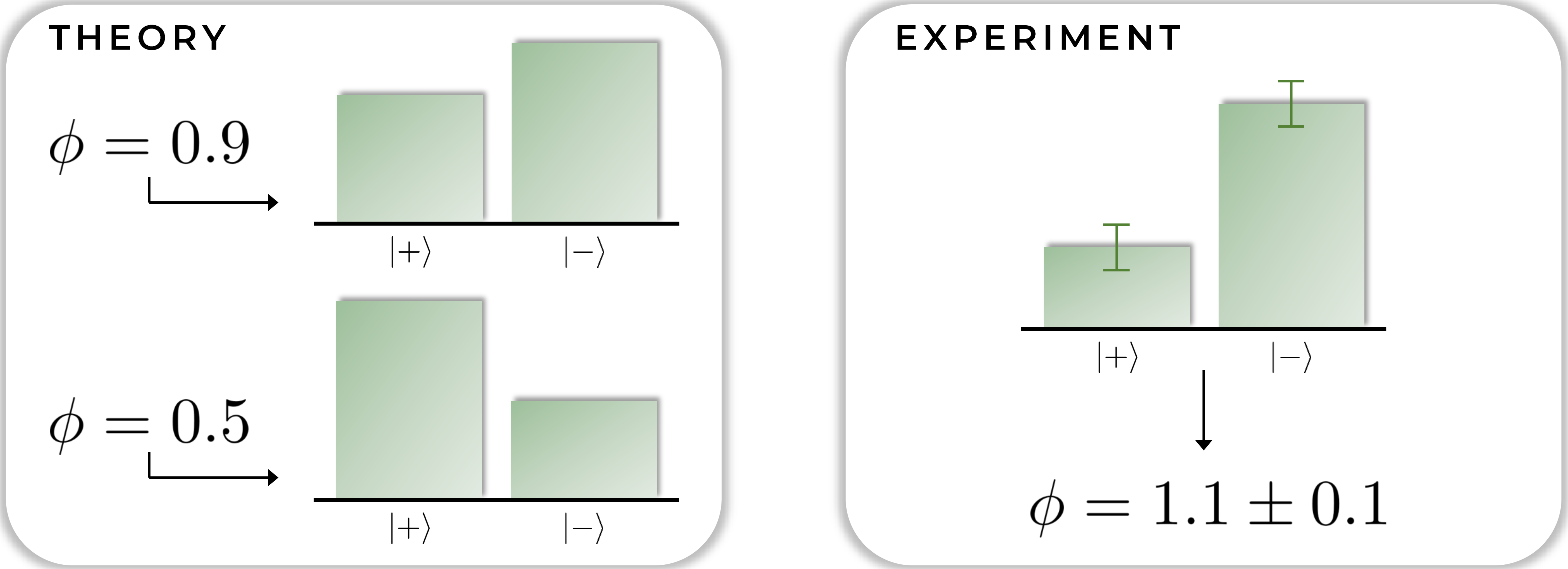






Gathering Intuition

$$|+\rangle \rightarrow e^{-i\phi Z} \rightarrow |+\rangle/|-\rangle$$



Measuring Performance

Measuring Performance

Task: Compute an **estimator** from the output distribution

$$p_l(\boldsymbol{\phi}) \;\rightarrow\; \hat{\boldsymbol{\varphi}} \colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

Measuring Performance

Task: Compute an **estimator** from the output distribution

$$p_l(\phi) \;\rightarrow\; \hat{\boldsymbol{\varphi}}\colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

Precision from n samples is limited by **Cramér-Rao bound**

$$\mathrm{Cov}(\hat{\boldsymbol{\varphi}}) \geq \frac{1}{n} I_{\boldsymbol{\phi}}^{-1}(\mathcal{M})$$

Measuring Performance

Task: Compute an **estimator** from the output distribution

$$p_l(\phi) \;\rightarrow\; \hat{\varphi} \colon \mathbb{E}\{\hat{\varphi}\} = \phi$$

Precision from n samples is limited by **Cramér-Rao bound**

$$\mathrm{Cov}(\hat{\varphi}) \geq \frac{1}{n} I_{\phi}^{-1}(\mathcal{M})$$

$$\mathrm{Tr}\{\mathrm{Cov}(\hat{\varphi})\} = \mathrm{MSE}(\hat{\varphi})$$

Measuring Performance

Task: Compute an **estimator** from the output distribution

$$p_l(\phi) \;\rightarrow\; \hat{\boldsymbol{\varphi}}\colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \phi$$

Precision from n samples is limited by **Cramér-Rao bound**

$$\mathrm{Cov}(\hat{\boldsymbol{\varphi}}) \geq \frac{1}{n} I_{\phi}^{-1}(\mathcal{M}) \geq \frac{1}{n} \mathcal{F}_{\phi}^{-1}$$

Measuring Performance

T

Fisher Information in Noisy Intermediate-Scale Quantum Applications

Johannes Jakob Meyer[1,2]

[1]Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany
[2]QMATH, Department of Mathematical Sciences, Københavns Universitet, 2100 København Ø, Denmark
28-03-2021

arXiv:2103.15191

$$\mathrm{Cov}(\hat{\varphi}) \geq \frac{1}{n} I_{\phi}^{-1}(\mathcal{M}) \geq \frac{1}{n} \mathcal{F}_{\phi}^{-1}$$

Recap #1

Recap #1

Sensing amounts to estimating the underlying physical parameters from a classical probability distribution

Recap #1

Sensing amounts to estimating the underlying physical parameters from a classical probability distribution

Estimation precision is limited by the Cramér-Rao bound

Recap #1

Sensing amounts to estimating the underlying physical parameters from a classical probability distribution

Estimation precision is limited by the Cramér-Rao bound

Attainble precision is quantified by the classical Fisher information

Recap #1

Sensing amounts to estimating the underlying physical parameters from a classical probability distribution

Estimation precision is limited by the Cramér-Rao bound

Attainble precision is quantified by the classical Fisher information

The quantum Fisher information bounds the attainable Fisher information

Recap #1

Sensing amounts to estimating the underlying physical parameters
from a classical probability distribution

Estimation precision is limited by the Cramér-Rao bound

Attainble precision is quantified by the classical Fisher information

The quantum Fisher information bounds the attainable Fisher information

→ Classical Fisher information should be used to judge sensing quality!

Optimal Metrology

Optimal Metrology

We need to find optimal probes and measurements

Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

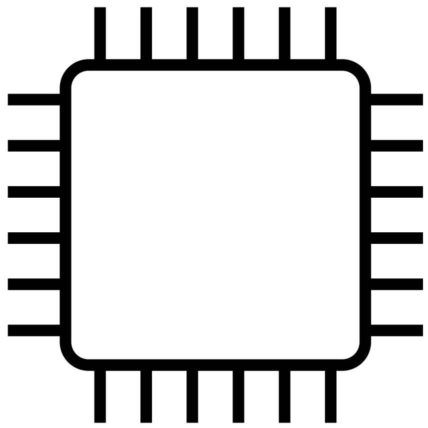
NISQ techniques come to the rescue:
use variational approaches

Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches

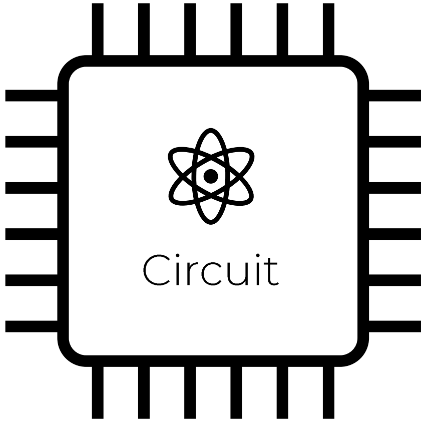


Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches

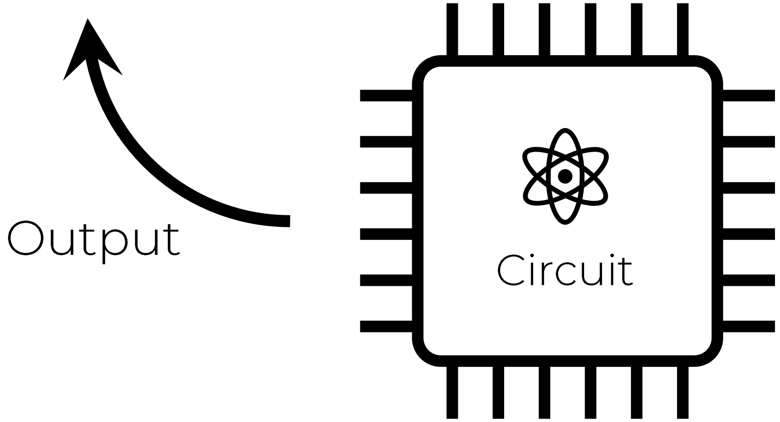


Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches

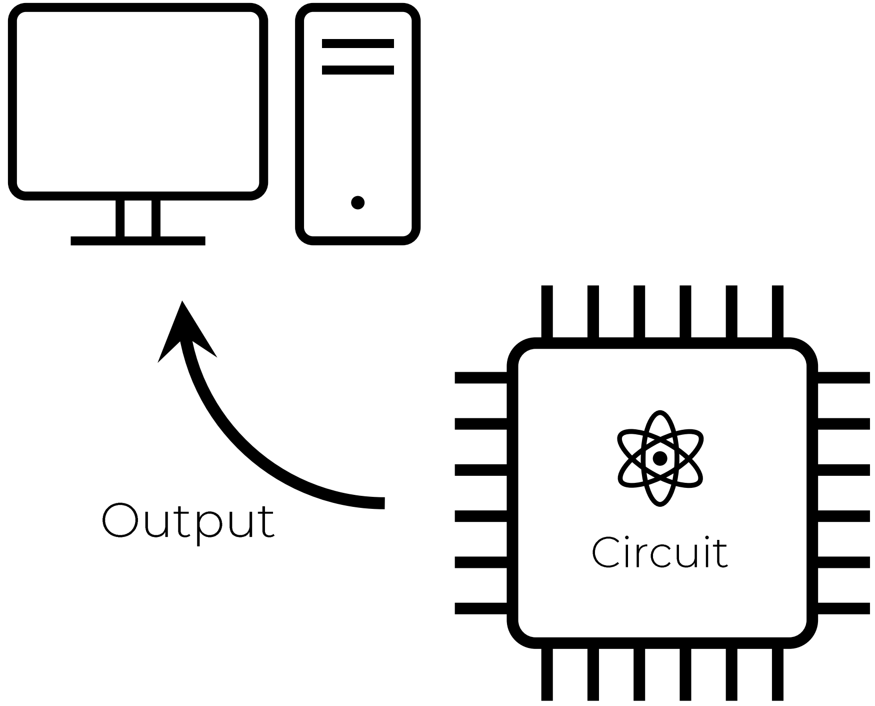


Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches

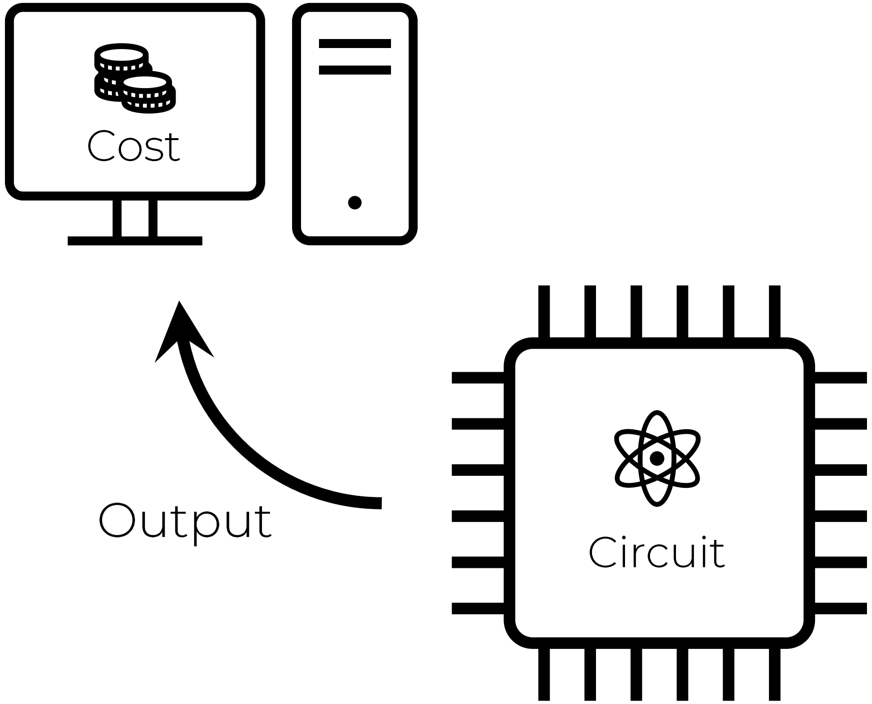


Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches

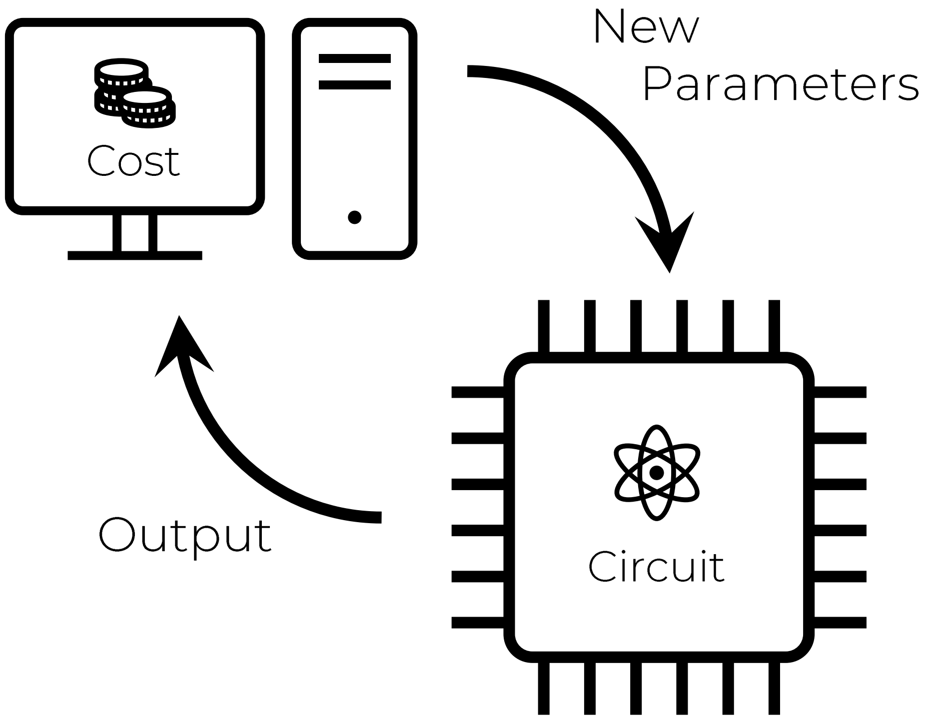


Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches

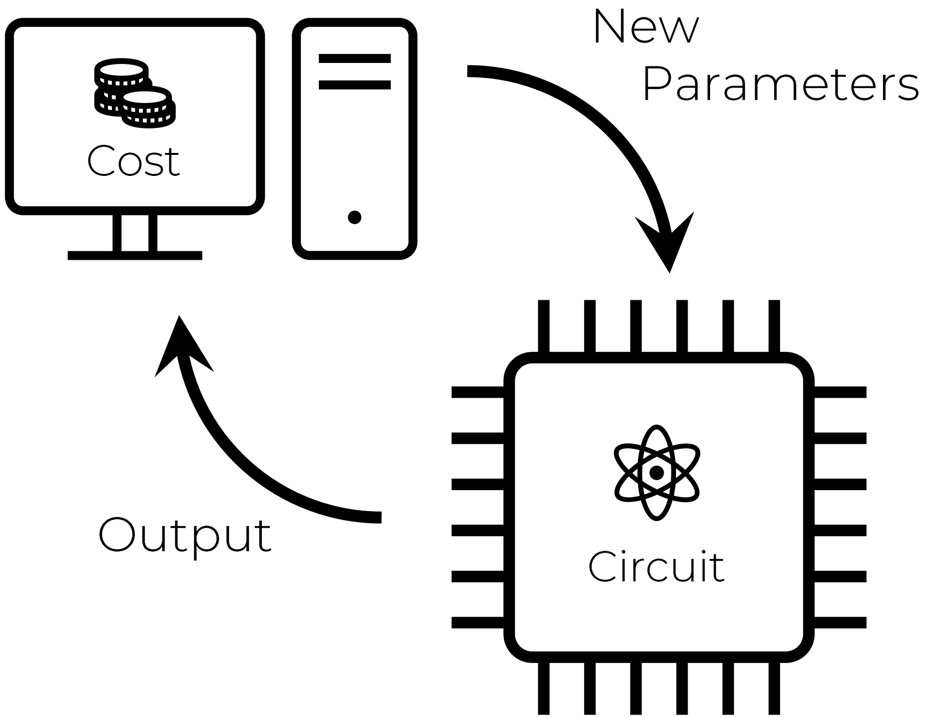


Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

NISQ techniques come to the rescue:
use variational approaches



Prior work[1,2] focused on single-parameter metrology and surrogates for the Quantum Fisher Information

[1]Kaubrügger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).

Cost Function

Cost Function

Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

Cost Function

Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{\phi}) \;\rightarrow\; I_{\boldsymbol{f}} = J^T I_{\boldsymbol{\phi}} J$$

Cost Function

Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{\phi}) \;\rightarrow\; I_{\boldsymbol{f}} = J^T I_{\boldsymbol{\phi}} J$$

Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

Cost Function

Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{\phi}) \;\rightarrow\; I_{\boldsymbol{f}} = J^T I_{\boldsymbol{\phi}} J$$

Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

$$\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{f}})\} \geq \frac{1}{n}\,\mathrm{Tr}\{W I_{\boldsymbol{f}}^{-1}\} = \frac{1}{n} C_W$$

Calculation of Fisher Information

Calculation of Fisher Information

Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_{l} \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$

Calculation of Fisher Information

Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_{l} \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$

Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2} \left[p_l \left(\boldsymbol{\phi} + \frac{\pi}{2} \boldsymbol{e}_j \right) - p_l \left(\boldsymbol{\phi} - \frac{\pi}{2} \boldsymbol{e}_j \right) \right]$$

[1]Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2]Banchi, Leonardo, and Gavin E. Crooks. *Quantum* 5 (2021): 386.

Recap #2

Recap #2

The classical Fisher information matrix is calculated from the output probabilities and their derivatives w.r.t. the physical parameters

Recap #2

The classical Fisher information matrix is calculated from the output probabilities and their derivatives w.r.t. the physical parameters

The derivatives can be calculated on the device via the parameter-shift rule

Recap #2

The classical Fisher information matrix is calculated from the output probabilities and their derivatives w.r.t. the physical parameters

The derivatives can be calculated on the device via the parameter-shift rule

The classical Fisher information matrix w.r.t. post-processed parameters can be computed using the post-processing‘s Jacobian

Recap #2

The classical Fisher information matrix is calculated from the output probabilities and their derivatives w.r.t. the physical parameters

The derivatives can be calculated on the device via the parameter-shift rule

The classical Fisher information matrix w.r.t. post-processed parameters can be computed using the post-processing‘s Jacobian

The cost function is obtained from a weighted trace of the Cramér-Rao bound

Have a Look at the Paper for

Have a Look at the Paper for

Multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

Have a Look at the Paper for

Multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

A parameter-shift rule for noise channels

Have a Look at the Paper for

Multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

A parameter-shift rule for noise channels

Details on the implementation of parameter-shift rules in experiments

Have a Look at the Paper for

Multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

A parameter-shift rule for noise channels

Details on the implementation of parameter-shift rules in experiments

Numerical experiments that showcase the performance of the approach

Take-Home Message

Variational methods on near-term quantum computers can be used to improve quantum sensors

Thank you for your attention!

Paper

Demo

Slides

Fisher Note

www.johannesjakobmeyer.com

@jj_xyz

The Algorithm Landscape

The Algorithm Landscape

Single Parameter

Multiparameter

The Algorithm Landscape

Single Parameter
Multiparameter

Kaubrügger et al.

COST FUNCTION
Spin Squeezing

STATE PREPARATION
Fixed Circuit

MEASUREMENT
Fixed

The Algorithm Landscape

Single Parameter | Multiparameter

Kaubrügger et al.

COST FUNCTION
Spin Squeezing

STATE PREPARATION
Fixed Circuit

MEASUREMENT
Fixed

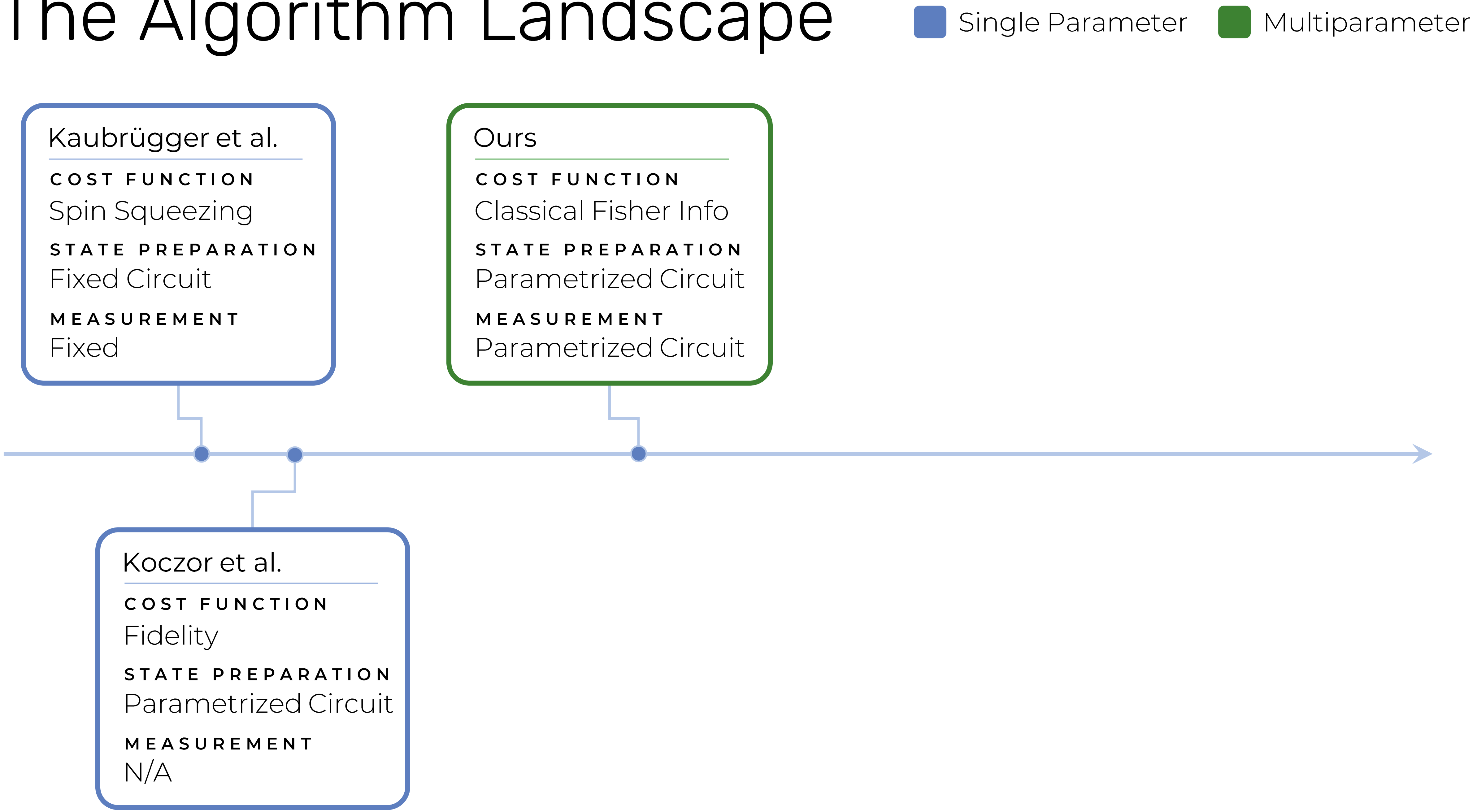
Koczor et al.

COST FUNCTION
Fidelity

STATE PREPARATION
Parametrized Circuit

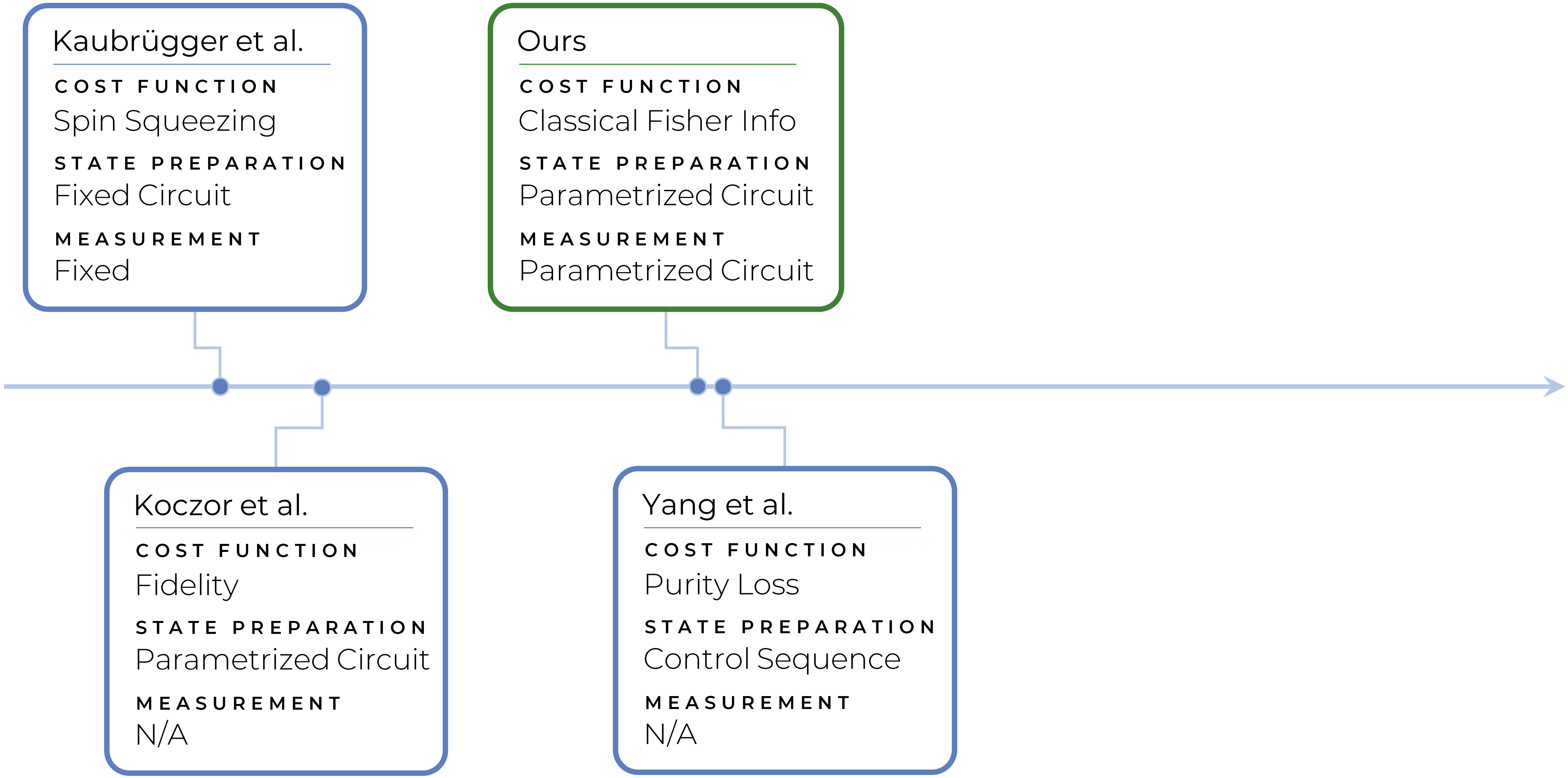
MEASUREMENT
N/A

The Algorithm Landscape

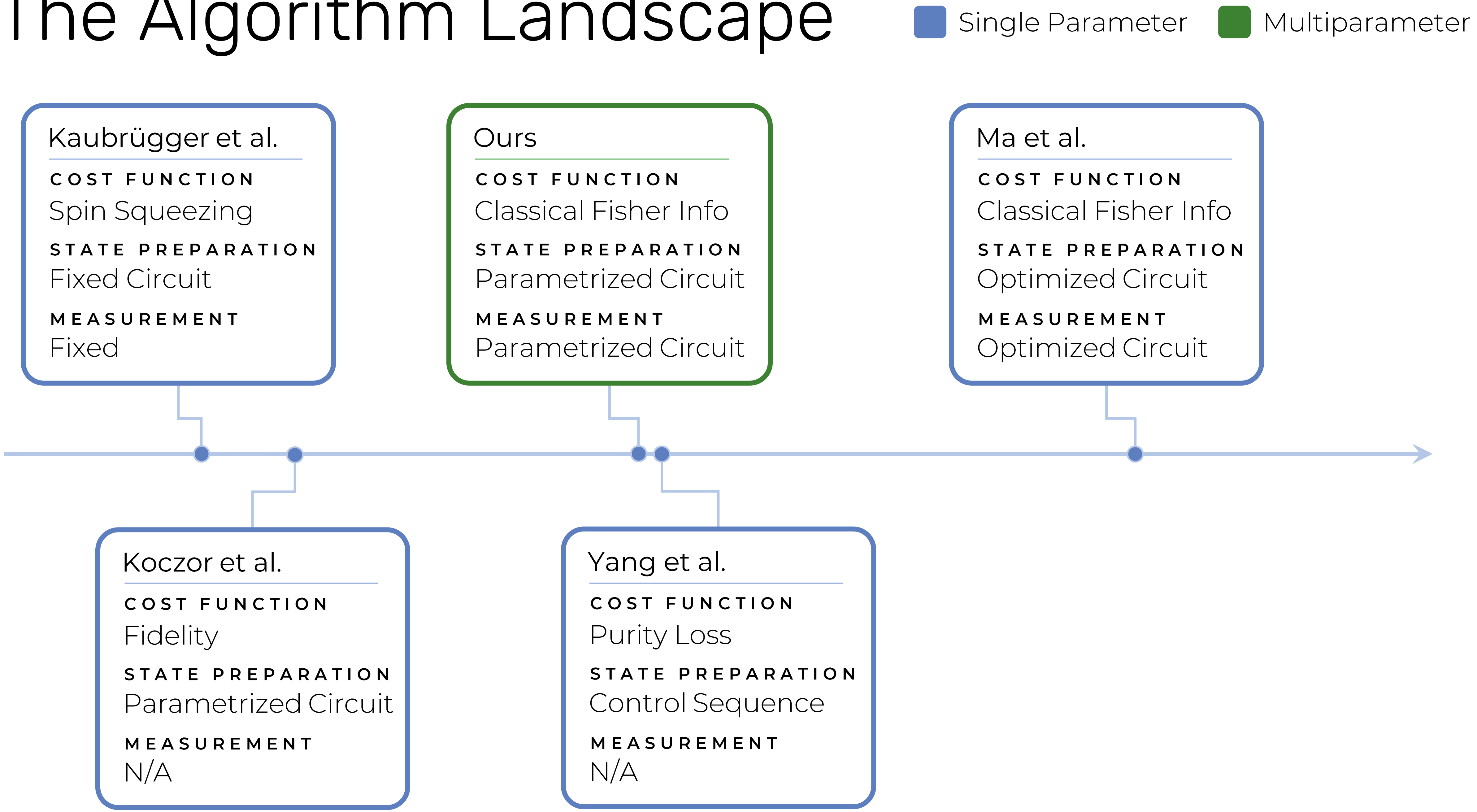


The Algorithm Landscape

Single Parameter
Multiparameter



The Algorithm Landscape



The Algorithm Landscape